**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans-

We have a normal distribution with = 45 and = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - )/ = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans - Mean = 38

SD = 6

Z score = (Value - Mean)/SD

a) Z score for 44 = (44 - 38)/6 = 1 => 84.13 %

=> People above 44 age = 100 - 84.13 = 15.87% ≈ 63 out of 400

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is FALSE

b) Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X - )/ = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees).

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans-

As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then

X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY ,

where X and Y are as defined above, i.e Z is linear combination of X and Y ,

then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D. print("""The two values of a and b, symmetric about the mean, are such that the probability of the random variable taking a value between them is 0.99:""",np.round(stats.norm.interval (0.99, loc = 100, scale = 20),1)) The two values of a and b, symmetric about the mean, are such that the probability of the random variable taking a value between them is 0.99: [ 48.5 151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: The 5TH Percentile of profit for the company is 17 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The Division #2 (Profit2 ~ N(7, 42) ) has a larger probability of making a loss in a given year.